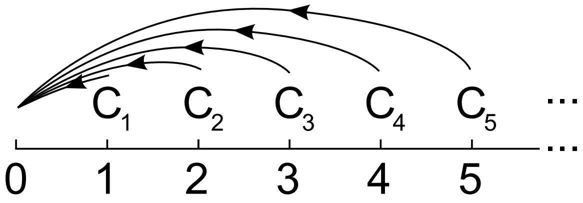
Lecture 2 – Team Activity – Stocks

Question 1: A stock is selling for $10 a share. It just paid its annual dividend of $2, so the next dividend will be paid in one year.

The dividend is not expected to change. What must be the required return on equity?

Answer:

$$PV(perpetuity\ with\ growth) = V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$



$$V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$10 = \frac{2}{r_{eff,annual} - 0}$$

$$r_{eff,annual} = \frac{2}{10} = 0.2 = 20\%$$

Question 2: A stock is selling for $10 a share. It just paid its annual dividend of $2, so the next dividend will be paid in one year.

The dividend is expected to grow at 4% pa. What must be the cost of equity?

Answer:

$$V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$10 = \frac{2 \times (1 + 0.04)}{r_{eff,annual} - 0.04}$$

$$10 \times \left(r_{eff,annual} - 0.04\right) = 2 \times (1 + 0.04)$$

$$r_{eff,annual} - 0.04 = \frac{2 \times (1 + 0.04)}{10}$$

$$r_{eff,annual} = \frac{2 \times (1 + 0.04)}{10} + 0.04$$

$$= 0.248 = 24.8\%$$

Question 3: A stock is selling for \$10 a share. It is just about to pay a dividend of \$2, so the next dividend will be paid at any moment (t=0).

The dividend is expected to grow at 4% pa. What must be the discount rate of equity?

Answer:

$$V_0 = PV(first\ \$2\ cash\ flow\ at\ t = 0) + PV(perpetuity)$$

$$V_0 = C_0 + \frac{C_1}{r_{eff} - g_{eff}}$$

$$10 = 2 + \frac{2 \times (1 + 0.04)}{r_{eff,annual} - 0.04}$$

$$8 = \frac{2 \times (1 + 0.04)}{r_{eff,annual} - 0.04}$$

$$8 \times \left(r_{eff,annual} - 0.04\right) = 2 \times (1 + 0.04)$$

$$r_{eff,annual} - 0.04 = \frac{2 \times (1 + 0.04)}{8}$$

$$r_{eff,annual} = \frac{2 \times (1 + 0.04)}{8} + 0.04$$

$$= 0.3 = 30\%$$

Question 4a: The NAB stock price is \$22.40. It **just paid** its **semi-annual** dividend of \$0.84 per share. The dividend is expected to grow by 2%, given as an effective 6-month rate. What is the stock's required return on equity, given as an effective annual rate?

$$V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$22.40 = \frac{0.84 \times (1 + 0.02)}{r_{eff,6mth} - 0.02}$$

$$22.40 \times (r_{eff,6mth} - 0.02) \ = 0.84 \times (1 + 0.02)$$

$$r_{eff,6mth} - 0.02 \ = \frac{0.84 \times (1 + 0.02)}{22.40}$$

$$r_{eff,6mth} \ = \frac{0.84 \times (1 + 0.02)}{22.40} + 0.02$$

$$= 0.05825$$

$$r_{eff,annual} \ = \left(1 + r_{eff,6mth}\right)^2 - 1$$

$$= (1 + 0.05825)^2 - 1$$

$$= 0.119893063$$

Question 4b: What is the total return, dividend yield, and capital return on the stock? You may express them all as effective 6 month rates.

Answer:

The total return is what we just found:

$$r_{total,eff\ 6mth} = 0.05825$$

The dividend yield (also called the income return) is:

$$r_{dividend,eff\ 6mth} = \frac{D_1}{V_0} = \frac{D_0 \times (1+g)}{22.40} = \frac{0.84 \times (1+0.02)}{22.40}$$

$$= 0.03825$$

The capital return is the growth in share price. The dividend growth rate is 2% per 6 months. The share price growth rate must be the same as the dividend growth rate.

$$r_{capital,eff\ 6mth} = growth\ rate\ in\ perpetual\ dividend = 0.02$$

Let's check that the total return is consistent with the dividend yield and the capital return:

$$r_{total,eff} = r_{income,eff} + r_{capital,eff}$$

$$0.05825 = 0.03825 + 0.02$$

$$0.05825 = 0.05825$$

Question 4c: How much do you expect the stock price and dividend to be in 3 years?

Answer:

The stock price and dividend will both grow by 2% every 6 months.

The dividend of $0.84, paid in 6 months, needs to be grown by 3 years (6 six-month periods):

$$D_{3\ years} = D_0 \times \left(1 + g_{eff,6mth}\right)^6$$

$$= 0.84 \times (1+0.02)^6$$

$$= \$0.945976432$$

The share price of $37.50, which is at the present time (t=0), needs to be grown by 3 years (6 six-month periods):

$$V_{3\,years} = V_0 \times \left(1 + g_{eff,6mth}\right)^6$$

$$= 22.40 \times (1 + 0.02)^6$$

$$= \$25.22603819$$

Question 5: The ROC (Roc Oil) stock price is $0.275. It doesn't pay dividends at all yet.

Your friend studies geology and knows about Roc Oil, and he says that in 3 years the company should be able to pay a constant semi-annual dividend of $0.05. So the first dividend payment will be at t=3 years. You estimate that the cost of equity is 12.36% pa, given as an effective annual rate. Using this information, what is your valuation of the stock?

Answer:

$$r_{eff,6mth} = \left(1 + r_{eff,annual}\right)^{\frac{1}{2}} - 1$$

$$= (1 + 0.1236)^{\frac{1}{2}} - 1 = 0.06$$

$$V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$V_5 = \frac{C_6}{r_{eff,6mth} - g_{eff,6mth}}$$

$$V_5 = \frac{0.05}{0.06 - 0}$$

$$V_5 = 0.83333$$

$$V_0 = \frac{0.83333}{(1 + 0.06)^5} = \$0.6227$$

Question 6: Google Inc. (GOOG on the NASDAQ) last traded at US$604.96 on 6/3/2012. On Google's Investor Relations website there is an FAQ which reads:

"Does Google pay a cash dividend?

No, we have never declared or paid a cash dividend nor do we expect to pay any dividends in the foreseeable future."

Since floating in 2004, Google has never paid a dividend nor completed a buy back. Do shareholders expect Google to pay a dividend or undertake a buy-back?

Answer:

Yes. Because the stock is worth $604.96, expected future dividends or buy backs must be significant. Eventually Google must pay a dividend or undertake a buy back, because if it doesn't ever return cash to shareholders then its shares would be worthless.

A similar case is Microsoft, which floated in 1986 and did not pay any dividends until 2003 (17 years later). High growth firms tend to re-invest their profits rather than pay dividends.

Lecture 2 – Team Activity – Debt

Question 1: What is the price of a \$1 million BAB maturing in 180 days if the yield is 10% pa?

Answer:

$$Price_{bill} = V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

$$= \frac{1{,}000{,}000}{\left(1 + 0.1 \times \frac{180}{365}\right)}$$

$$= 953{,}002.61$$

Question 2: A BAB with a face value of \$100,000 is trading at a price of \$99,000. It matures in 60 days. What is the yield on the bill?

Answer:

$$Price_{bill} = V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

$$99{,}000 = \frac{100{,}000}{\left(1 + r_{simple} \times \frac{60}{365}\right)}$$

$$99{,}000 \times \left(1 + r_{simple} \times \frac{60}{365}\right) = 100{,}000$$

$$1 + r_{simple} \times \frac{60}{365} = \frac{100{,}000}{99{,}000}$$

$$r_{simple} \times \frac{60}{365} = \frac{100{,}000}{99{,}000} - 1$$

$$r_{simple} = \left(\frac{100{,}000}{99{,}000} - 1\right) \times \frac{365}{60} = 0.061447811 = 6.1448\%$$

Question 3: If a zero-coupon bond is issued at par, what must its yield be equal to?

Answer: Yields would have to be zero. This is because a par bond's yield must equal its coupon rate. Zero yields are very odd. Zero coupon bonds are generally discount securities since yields are usually positive.

Question 4: A bond maturing in 10 years that has a coupon rate of 12%, paid **semi-annually**. The bond's yields is currently 10% pa. The face value of the bond is $100. What is its price?

Answer:

$$C_1 = face\ value \times \frac{coupon\ rate}{2}$$

$$= 100 \times \frac{0.12}{2} = \$6$$

$T = 10\ years * 2 = 20$ semi-annual periods

$$r_{eff,semi-annual} = \frac{r_{APR,comp\ semi-annually}}{2}$$

$$= \frac{0.1}{2} = 0.05$$

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{6}{0.05}\left(1 - \frac{1}{(1 + 0.05)^{20}}\right) \quad + \quad \frac{100}{(1 + 0.05)^{20}}$$

$$= 74.7733 + 37.6889$$

$$= 112.4622$$

The price of 112.4622 is more than the face value of 100, as it should be, since the coupon rate is higher than the yield.

Question 5: A bond maturing in 10 years yields 10% pa. The coupon rate is 8%, paid **annually**. The face value of the bond is \$100. What is its price?

Answer:

$$C_1 = face\ value \times coupon\ rate$$

$$= 100 \times 0.08 = \$8$$

$$T = 10\ years$$

$$r_{eff,annual} = r_{APR,comp\ annually}$$

$$= 0.1$$

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{8}{0.1}\left(1 - \frac{1}{(1 + 0.1)^{10}}\right) \quad + \quad \frac{100}{(1 + 0.1)^{10}}$$

$$= 49.1565 + 38.5543$$

$$= 87.7109$$

The price of 87.7109 is less than the face value of 100, as it should be, since the coupon rate is less than the yield.

Question 6a: A 3 year 6% semi-annual coupon bond with a face value of $100 is issued at par. What is its price? What is its yield?

Answer: Since the bond is issued at par, the price is equal to the $100 face value and the yield is equal to the 6% coupon rate. The 6% yield is an APR compounding semi-annually.

Question 6b: Exactly one year later, just after the second coupon was paid, yields have fallen to 5% pa. What is the new bond price?

Answer: Let's take short-cuts and do the coupon, time and effective rate calculation in the bond formula. Note that there are 4 semi-annual coupons left to be paid.

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{100 \times 0.06/2}{0.05/2}\left(1 - \frac{1}{(1 + 0.05/2)^4}\right) + \frac{100}{(1 + 0.05/2)^4}$$

$$= 11.2859 + 90.5951$$

$$= 101.8810$$

The price of The price of 101.8810 is more than the face value of 100, as it should be, since the coupon rate is more than the yield.

Question 6c: A 3 year 6% semi-annual coupon bond with a face value of \$100 was issued at par 2.5 years ago. Now, six months before maturity, only the last coupon and principal are owing. The price of the bond is \$98. What is its yield at this time?

Answer: The last coupon and principal payment are paid at the same time at maturity which is in 6 months. The bond price is the present value of these cash flows.

The coupon rate doesn't change, so the last coupon will be \$3.

$$Price_{bond} = \frac{last\ coupon + principal}{\left(1 + r_{eff}\right)^T}$$

$$98 = \frac{3 + 100}{\left(1 + r_{eff,6mth}\right)^1}$$

$$98 = \frac{103}{1 + r_{eff,6mth}}$$

$$98 \times (1 + r_{eff,6mth}) = 103$$

$$1 + r_{eff,6mth} = \frac{103}{98}$$

$$r_{eff,6mth} = \frac{103}{98} - 1 = 0.05102$$

$$r_{APR,comp\ 6mth} = r_{eff,6mth} \times 2$$

$$= 0.05102 \times 2$$

$$= 0.10204 = 10.204\%$$

So yields increased from 6% pa to 10.204% pa. This makes sense since the bond price fell, so yields must have increased.

The yield is the discount rate of the bond's cash flows so if the bond price fell (present value fell), then it must be because the yield (discount rate) increased.

Question 7a: A bond trading in the **EU** has the following details:
Maturity: 2yrs
Coupon rate: 5%**, paid annually**
Yield: 10%
Face value: 100
Find the bond price. Note that **European bonds pay annual coupons**.

Answer: Since European bonds pay annual coupons, yields are quoted as APR's compounding per year, which is the same as an Effective Annual Rate (EAR).

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{100 \times 0.05}{0.1}\left(1 - \frac{1}{(1 + 0.1)^2}\right) + \frac{100}{(1 + 0.1)^2}$$

$$= 8.67768595 + 82.6446281$$

$$= 91.3223$$

The price of 91.3223 is less than the face value of 100, as it should be, since the coupon rate is less than the yield. This is a discount bond.

Question 7b: A bond trading in the **US** has the following details:
Maturity: 2yrs
Coupon rate: 5%, paid semi-annually
Yield: 10%
Face value: 100
Find the bond price. Note that **US (and Australian) bonds pay semi-annual coupons**.

Answer: Since US bonds pay semi-annual coupons, yields are quoted as APR's compounding every six months, which is two times the effective 6 month rate.

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{100 \times 0.05/2}{0.1/2}\left(1 - \frac{1}{(1 + 0.1/2)^4}\right) + \frac{100}{(1 + 0.1/2)^4}$$

$$= 8.86487626 + 82.27024748$$

$$= 91.1351$$

The price of 91.1351 is less than the face value of 100, as it should be, since the coupon rate is less than the yield. This is a discount bond.

Question 7c: Compare the US bond to the EU bond in the previous question. Which bond promises a higher yield, or do they promise the same yield? Hint: APR's of different compounding periods cannot be compared.

Answer: While both bonds have a yield of 10%, the US bond yield compounds semi-annually and the European bond compounds annually, so they can't be compared.

Let's convert the US bond yield into an effective annual rate (EAR), which is how the European bond yield is quoted.

$$r_{US,eff,annual} = \left(1 + \frac{r_{US,APR\ comp\ 6mth}}{2}\right)^2 - 1$$

$$= \left(1 + \frac{0.1}{2}\right)^2 - 1 = 0.1025$$

This is more than the European bond yield of 0.1, therefore the US bond offers the higher promised yield.