Lecture 1 – Team Activity

Question 1: A stock was bought for \$10 and sold one month later for \$10.50.

a) What is the effective monthly rate of return?

$$V_0 = \frac{V_t}{(1+r)^t}$$

$$10 = \frac{10.5}{\left(1 + r_{eff,monthly}\right)^1}$$

$$r_{eff,monthly} = \frac{10.5}{10} - 1 = 0.05 = 5\%$$

b) What is the APR compounding per month?

$$r_{APR,comp\ monthly} = 12 \times r_{eff,monthly}$$

$$= 12 \times 0.05$$

$$= 0.6 = 60\%$$

c) What is the effective annual rate of return?

$$r_{eff,annual} = \left(1 + r_{eff,monthly}\right)^{12} - 1$$

$$= (1 + 0.05)^{12} - 1$$

$$= 0.7959 = 79.59\%$$

Question 2: A project costs \$100 now and \$50 in 10 years. In 3 years it will pay back \$40, and every year after that for the next 5 years it will pay back \$20. The effective annual interest rate is 10%. What is the present value of the project? The below timeline shows the cash flows.

-100			40	20	20	20	20	20		-50
0	1	2	3	4	5	6	7	8	9	10

$$V_0 = -100 + \frac{40}{(1+0.1)^3} + \frac{\frac{20}{0.1}\left(1 - \frac{1}{(1+0.1)^5}\right)}{(1+0.1)^3} + \frac{-50}{(1+0.1)^{10}}$$

$$= -100 + 30.0526 + 56.9615 - 19.2772$$

$$= \mathbf{-32.2631}$$

This is just an addition of present values.

The complicated looking expression $\frac{20}{0.1}\left(1 - \frac{1}{(1+0.1)^5}\right)$ is the annuity of five \$20 payments, which is discounted by another 3 years because the annuity formula gives a value one year before the first \$20 cash flow in year 4, which is 3 years away. Alternatively, to try explaining it using maths:

$$V_{0,annuity} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$V_{3,annuity} = \frac{C_4}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$= \frac{20}{0.1}\left(1 - \frac{1}{(1+0.1)^5}\right) = 75.8157$$

$$V_{0,annuity} = \frac{75.8157}{(1+0.1)^3} = 56.9615$$

Question 3: You want to buy a \$300,000 apartment. You haven't saved a deposit. The bank offers you a fully amortising mortgage with a term of 30 years and an interest rate of 6% pa. How much will your monthly payments be?

Since it is a fully amortising mortgage, at maturity the whole principal will already be paid off, so the annuity formula is all that is needed.

Since mortgages are paid monthly, the 6% interest rate would be an APR compounding per month. The APR must be converted to an effective monthly rate so we can discount the monthly payments:

$$r_{eff,monthly} = \frac{r_{APR,comp\ monthly}}{12} = \frac{0.06}{12} = 0.005 = 0.5\%$$

The number of monthly periods until maturity in 30 years is:

$t = 30 \times 12 = 360$ months

Using the annuity formula:

$$V_0 = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$300{,}000 = \frac{C_1}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right)$$

$$C_1 = 300{,}000 \div \left(\frac{1}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right)\right)$$

$$= 300{,}000 \div 166.7916144$$

$= \$1{,}798.6516$ at the end of each month.

Question 4: A credit card advertises an interest rate of 24%.

Note that credit cards are paid monthly so the interest rate is quoted as an Annualised Percentage Rate (APR) compounding per month.

a) Find the effective monthly rate.

$$r_{eff,monthly} = \frac{r_{APR,comp\ monthly}}{12} = \frac{0.24}{12} = 0.02$$

b) Find the effective annual rate.

$$r_{eff,annual} = \left(1 + r_{eff,mthly}\right)^{12} - 1 = \left(1 + \frac{0.24}{12}\right)^{12} - 1 = 0.2682$$

c) Find the effective 6 month rate.

$$r_{eff,6mth} = \left(1 + r_{eff,mthly}\right)^{6} - 1 = \left(1 + \frac{0.24}{12}\right)^{6} - 1 = 0.1262$$

d) Find the effective quarterly rate.

$$r_{eff,qtrly} = \left(1 + r_{eff,mthly}\right)^{3} - 1 = \left(1 + \frac{0.24}{12}\right)^{3} - 1 = 0.0612$$

e) Find the Annualised Percentage Rate (APR), compounding every 6 months ($r_{APR,comp\ per\ 6mths}$).

$$r_{APR,comp\ per\ 6mths} = 2 \times r_{eff,6mth}$$

$$= 2 \times \left(\left(1 + \frac{0.24}{12}\right)^{6} - 1\right) = 0.2523$$

f) Find the APR compounding per day ($r_{APR,comp\ daily}$). Assume 30 days in a month and 360 days in a year.

$$r_{APR,comp\ daily} = 360 \times r_{eff,daily}$$

$$= 360 \times \left(\left(1 + \frac{0.24}{12}\right)^{\frac{1}{30}} - 1 \right) = 0.2377$$

Question 5: A bond is advertised with a coupon rate of 7%, paid semi-annually. The yield of the bond is 6%.

Note that the bond pays semi-annual coupons so the yield is quoted as an Annualised Percentage Rate (APR) compounding every 6 months.

a) Find the effective six-month rate.

$$r_{eff,6mth} = \frac{r_{APR,comp\ per\ 6\ mths}}{2} = \frac{0.06}{2} = 0.03$$

b) Find the effective annual rate.

$$r_{eff,annual} = \left(1 + r_{eff,6mth}\right)^2 - 1 = \left(1 + \frac{0.06}{2}\right)^2 - 1 = 0.0609$$

c) Find the effective monthly rate.

$$r_{eff,monthly} = \left(1 + r_{eff,6mth}\right)^{\frac{1}{6}} - 1$$

$$= \left(1 + \frac{0.06}{2}\right)^{1/6} - 1 = 0.004938622$$

d) Find the effective quarterly rate.

$$r_{eff,qtrly} = \left(1 + r_{eff,6mth}\right)^{\frac{1}{3}} - 1$$

$$= \left(1 + \frac{0.06}{2}\right)^{1/3} - 1 = 0.009901634$$

e) Find the Annualised Percentage Rate (APR), compounding every week. Assume 52 weeks per year. ($r_{APR,comp\ weekly}$).

$$r_{APR,comp\ weekly} = 52 \times r_{eff,weekly}$$

$$= 52 \times \left(\left(1 + \frac{0.06}{2}\right)^{1/26} - 1\right) = 0.059151222$$

f) Find the APR compounding per day. Assume 30 days in a month and 360 days in a year.

$$r_{APR,comp\ daily} = 360 \times r_{eff,daily}$$

$$= 360 \times \left(\left(1 + \frac{0.06}{2}\right)^{\frac{1}{180}} - 1\right) = 0.059122459$$