MACQUARIE UNIVERSITY
SYDNEY ~ AUSTRALIA

Financial Management
ACCG253

Lecture 8

WACC, Leverage and M&M

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The Cost of Equity (r_e)

The cost of equity (r_e) is the rate of return that equity holders deserve for their level of risk. It has many names including the required return on equity, shareholders' cost of capital, and stock-holder's required return.

There are two methods to find the cost of equity.

We can use the Dividend Discount Model (DDM):

$$r_e = \frac{C_1}{P_0} + g$$

or the Capital Asset Pricing Model (CAPM):

$$r_e = r_f + B_e(r_m - r_f)$$

DDM to find the Cost of Equity

We can find the cost of equity using the Dividend Discount Model, also known as Gordon's Growth Model or the perpetuity with growth formula,

$$\boldsymbol{P_0 = \frac{C_1}{r_e - g}}$$

C_1 = cash flow received at $t = 1$. The cash flows go on forever, but grow by g every period. For stocks, the cash flow is the dividend.

g = effective growth rate of the dividend C_1 per period. It is also the capital return (price increase) of the stock.

r_e = effective cost of equity over a single period. It is the total return of the stock.

After re-arranging the equation to make r_e the subject, we get:

$$\boldsymbol{r_e = \frac{C_1}{P_0} + g}$$

Note that this is the familiar formula that separates total return into its income and capital components, but applied to an equity security (a stock or share).

$$r_{total} = r_{income} + r_{capital}$$

This makes sense since g is the capital return and $\frac{C_1}{P_0}$ is the dividend yield which is a stock's form of income return.

CAPM (or SML) to find the Cost of Equity

$$\boldsymbol{r_e = r_f + B_e(r_m - r_f)}$$

Where:

r_e = effective total return of the stock 'e'.

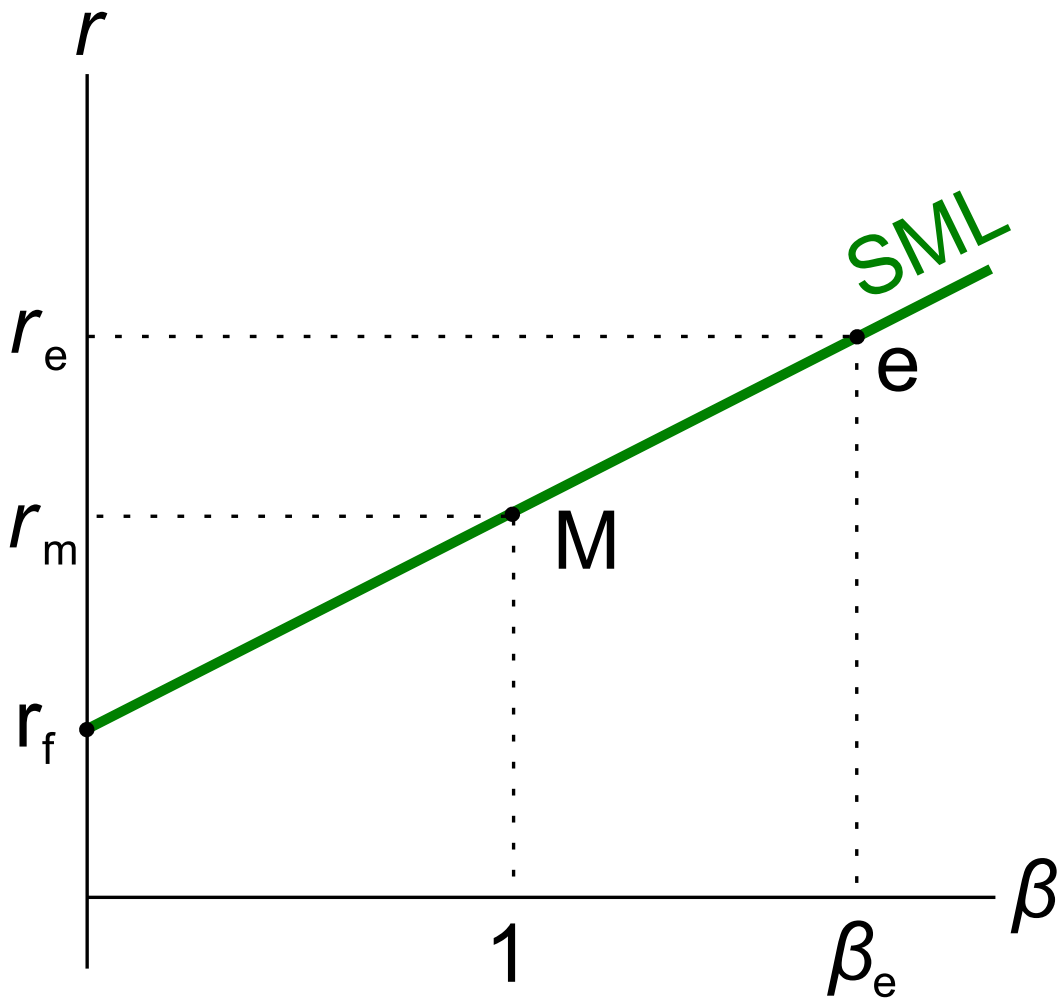
B_e = beta of the stock 'e'. The beta is a measure of systematic risk, defined as $B_e = \frac{cov(r_e, r_m)}{var(r_m)}$.

r_f = effective total return of the risk free asset (government treasury bonds).

r_m = effective total return of the market portfolio (stock index for example the ASX200 (Australia) or S&P500 (US)).

This method of finding the cost of equity is also called the SML (Security Market Line) method.

This is because we are finding r_e on the SML using the stock's beta B_e.



Calculation Example: Cost of Equity

Question: Find the firm's cost of equity using:

(i) the DDM and

(ii) the CAPM or SML

with the information below:

The firm's stock price is $20,

Beta of equity is 1.5,

Market return is 10% p.a.,

Treasury bonds yield 5% p.a.,

The stock will pay its next annual dividend of $2.50 in one year, which grows at a rate of 2% p.a.. All rates are effective pa.

Answer:

(i) Using the Dividend Discount Model (DDM):

$$r_{e,DDM} = \frac{D_1}{P_0} + g$$

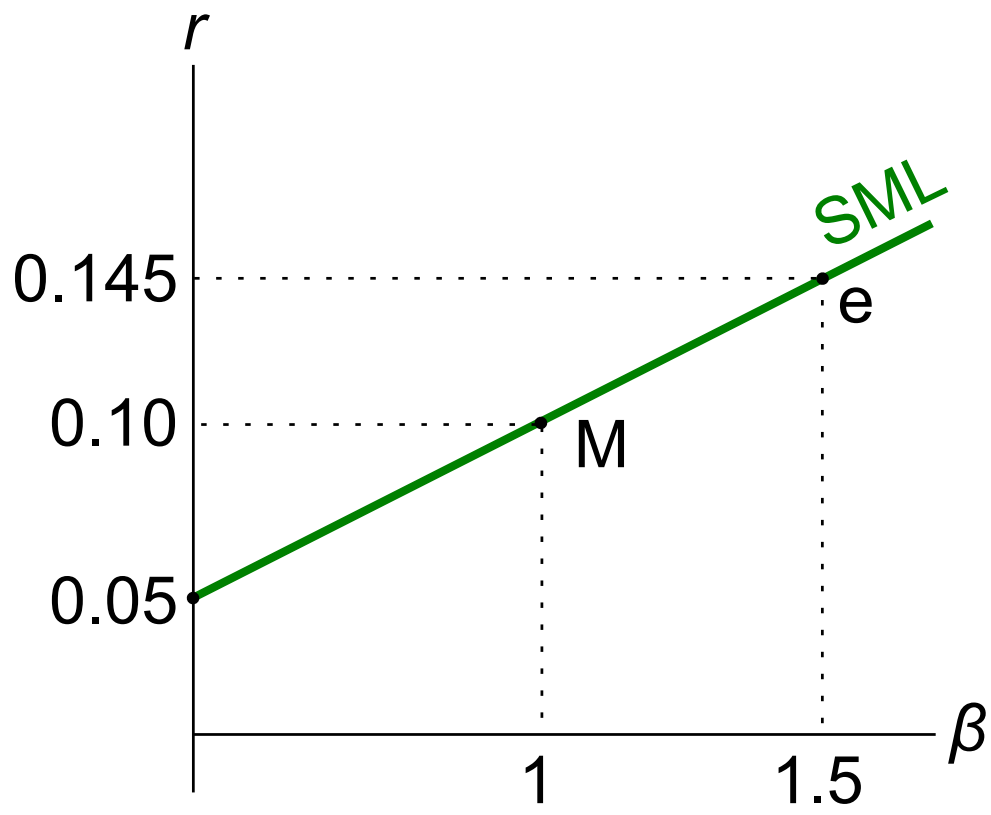
$$= \frac{2.50}{20} + 0.02 \quad = 0.145$$

(ii) Using the Capital Asset Pricing Model (CAPM):

$$r_{e,CAPM} = r_f + B_e(r_M - r_f)$$

$$= 0.05 + 1.5 \times (0.1 - 0.05)$$

$$= 0.125$$



In theory, they should both be the same. They are only different because our input numbers are inaccurate, and/or because the assumptions of the models are violated.

For example, the DDM assumes dividends grow forever at a constant rate which is obviously not going to happen in reality. The (static) CAPM assumes that the beta doesn't change which is also silly.

In practice, an arbitrary weighted average of the two might be used, weighted according to which one you think is more accurate and suitable for the project being valued.

The Cost of Debt (r_d)

The cost of debt (r_d) is just the yield on debt, for example the yield on the firm's bonds.

The cost of debt is also known as the required return on debt, debt-holders' cost of capital, debt-holder's required return, or total return on debt.

Finding the cost of debt (the yield) requires a financial calculator or a spreadsheet and the solver function. The exception is the more simple zero-coupon bonds whose yields can easily be found using basic algebra and an ordinary calculator.

Revision:

$$NI = (Rev - COGS - Depr - IntExp).(1 - t_c)$$

$$CFFA = NI + Depr - CapEx - \uparrow NWC + IntExp$$

$$CFFA = \quad CF\ to\ equity\ holders \quad + CF\ to\ creditors$$

$$V = E + D$$

Weighted Average Cost of Capital (WACC)

$$WACC_{before-tax} = r_d . \frac{D}{V} + r_e . \frac{E}{V}$$

$$WACC_{after-tax} = r_d . (1 - t_c) . \frac{D}{V} + r_e . \frac{E}{V}$$

The weighted average cost of capital, the WACC, is the cost of equity (r_e) and the cost of debt (r_d) weighted by the proportion of equity (E) and debt (D) used to finance the firm's assets (V). Note that E, D and V are all *market* values not *book* values.

The value of a firm (V) is the Cash Flow From Assets (CFFA) discounted by the firm's WACC. The only complicating factor is taxes...

The Benefit of Debt: Interest Tax Shields

Interest expense is looked upon by the tax office as a cost of doing business, so it is tax-deductible. That's why it is subtracted from NI before tax is paid:

$$NI = (Rev - COGS - Depr - \boldsymbol{IntExp}).(1 - t_c)$$

However, we know that interest is just a form of income to debt holders, just as dividends are income to equity holders. In a perfect world with no taxes or transaction costs we wouldn't care if we financed our project with debt or equity. But since interest is tax-deductible and dividends aren't, debt is tax-advantaged.

Quantifying the Interest Tax Shield

$$NI = (Rev - COGS - Depr - \boldsymbol{IntExp}).(1 - t_c)$$

$$CFFA = NI + Depr - CapEx - \uparrow NWC + \boldsymbol{IntExp}$$

After substituting the NI equation into CFFA, and then expanding and collecting like terms, the following can be shown:

$$CFFA = (Rev - COGS).(1 - t_c) + Depr.t_c - CapEx - \uparrow NWC + \boldsymbol{IntExp}.\boldsymbol{t_c}$$

That last term, $\boldsymbol{IntExp}.\boldsymbol{t_c}$, is the tax shield per year. It is the tax saving from paying interest on debt.

Note that $\boldsymbol{IntExp} = \boldsymbol{D}.\boldsymbol{r_d}$. where D is the value of the firm's debt.

Calculation Example: Interest Tax Shield

Just Jeans Group Income Statement for period ending 26 July 2008	
Net sales	822
COGS	717
Depreciation	24
EBIT	81
Interest expense	11
Taxable income	70
Taxes	21
Net income	49

Just Jeans Group Balance Sheet as at 26 July	2008	2007
Current A	92	105
Non-current A	195	178
Total A	287	259
Current L	208	72
Non-current L	22	134
Owners Equity	57	53
Total L and OE	287	259

Note: all figures are given in millions of dollars ($m).

Question: Find the yearly interest tax shield.

Answer: The easy way is to use the formula:

Interest tax shield per year $= \boldsymbol{IntExp.t_c}$

$$= \mathbf{11}\boldsymbol{m} \times \mathbf{0.3} = \$\mathbf{3.3}\boldsymbol{m}$$

So $3.3m is the tax saving from paying interest on debt. If the firm didn't have this debt then it would have a lower CFFA and the value of the firm would be less.

Valuation with Debt

If a firm has no debt, we say it is 'unlevered' or 'all-equity'.

Let V_U signify the unleveraged value, and V_L signify the leveraged value. From the equation $V = D + E$,

$V_L = D + E_L$, and

$V_U = E_U$, since debt is zero in an unlevered firm.

Also, the value of the levered firm must equal the value of the unlevered firm plus the present value of the interest tax shields:

$V_L = V_U + PV(interest\ tax\ shields)$

The 'Textbook Method' of Valuation: $CFFA_{Unlevered}$ and $WACC_{after-tax}$

The easiest way to value a project is to calculate its cash flows from assets as if it is all-equity financed, so there is no debt and no interest expense. That is, find $CFFA_{Unlevered}$.

Then discount these unlevered cash flows using the firm's after tax weighted average cost of capital, $WACC_{after-tax}$.

This will give the correct value of the firm including the present value of the interest tax shields

This method takes the interest tax shield into account in the **discount rate** rather than the **cash flow**.

There are two key assumptions:

- the firm has a target debt-to-assets ratio (D/V) or debt-to-equity ratio (D/E) that it sticks to, and
- the project is of similar risk to the rest of the business.

It might seem curious that the tax shield benefit is included in the project's value when we used the CFFA excluding debt.

The reason is because the interest tax shield benefit is included in the discount rate (the after-tax WACC) rather than the cash flow (the unlevered CFFA). The cost of debt in the after-tax WACC, r_D, is being multiplied by $(1 - t_c)$ which reduces the cost of debt by the amount of the tax shield. The lower discount rate (WACC) makes for a higher value (V), and this increase in value is due to the interest tax shields.

Calculation Example: Firm Valuation

Question 1: A firm has a target debt-to-assets ratio of 25%.

The cost of equity is 10%. The cost of debt is 5%.

The tax rate is 30%.

The firm's unlevered cash flow from assets is $10m each year which is constant and earned at the end of every year forever. Note that since this is the unlevered CFFA, interest expense and therefore tax shields are zero.

Find the value of the levered firm (V_L).

Answer: We are already given the unlevered CFFA, so the next step is to find the after-tax WACC:

$$WACC_{after-tax} = r_d.(1 - t_c).\frac{D}{V} + r_e.\frac{E}{V}$$

$$= 0.05 \times (1 - 0.3) \times 0.25 + 0.1 \times 0.75$$

$$= 0.08375$$

Since the positive cash flows go on forever, we will use the perpetuity formula ($V_0 = \frac{C_1}{r-g}$) to find the value of the levered firm:

$$V_L = PV(CFFA_{Unlevered}, discounted\ by\ WACC_{after-tax})$$

$$V_L = \frac{10m}{0.08375} = \$119.403m$$

The 'harder method' of Valuation: $CFFA_{Levered}$ and $WACC_{before-tax}$

The other (harder) way to value a project is to calculate its cash flows from assets including debt, and then to discount these using the WACC before tax.

This method takes the interest tax shield into account in the cash flow rather than the discount rate. This method also assumes that:

- the firm has a target debt-to-assets ratio (D/V) or debt-to-equity ratio (D/E) that it sticks to, and
- the project is of similar risk to the rest of the business.

Calculation Example: Firm Valuation

In the previous question the firm had a levered value (V_L) of \$119.403m.

Question 2: Show that the levered value of the firm using the harder method gives the same result.

Answer:

$$WACC_{before-tax} = r_d . \frac{D}{V} + r_e . \frac{E}{V}$$

$$= 0.05 \times 0.25 + 0.1 \times 0.75$$

$$= 0.0875$$

We need to find the levered CFFA. This depends on the yearly tax shield which depends the amount of debt D. Since the debt-to-assets ratio is 25%,

$$\frac{D}{V_L} = 0.25$$

$$D = 0.25 \times V_L$$

Let's substitute this into the below equation,

$$\begin{aligned} CFFA_{levered} &= CFFA_{\boldsymbol{un}levered} + IntExp \times t_c \\ &= CFFA_{\boldsymbol{un}levered} + D \times r_d \times t_c \\ &= CFFA_{\boldsymbol{un}levered} + 0.25 \times V_L \times r_d \times t_c \\ &= 10m + 0.25 \times V_L \times 0.05 \times 0.3 \end{aligned}$$

Since the positive cash flows go on forever, we will use the perpetuity formula ($V_0 = \frac{C_1}{r-g}$) to find the value of the levered firm:

$$V_L = PV(CFFA_{Levered}, discounted\ by\ WACC_{before-tax})$$

$$V_L = \frac{10m + 0.25 \times V_L \times 0.05 \times 0.3}{0.0875}$$

$$V_L \times 0.0875 - 0.25 \times V_L \times 0.05 \times 0.3 = 10m$$

$$V_L \times (0.0875 - 0.25 \times 0.05 \times 0.3) = 10m$$

$$V_L = \frac{10m}{(0.0875 - 0.25 \times 0.05 \times 0.3)}$$

$$= \$119.4030m$$ Which is the same as before!

Question 3: Find the present value of the perpetual stream of tax shields.

Answer:

$$interest\ tax\ shield\ per\ year = IntExp \times t_c$$

$$= D \times r_d \times t_c$$

Now to find the present value of the interest tax shields we use the perpetuity formula,

$$PV(interest\ tax\ shields) = \frac{interest\ tax\ shield\ per\ year}{r_d}$$

$$= \frac{D \times r_d \times t_c}{r_d} = D \times t_c = 29.8508m \times 0.3 = \$8.955m$$

Question 4: What would be the value of the firm if it was all-equity financed? In other words, the firm replaced its debt with equity?

Answer:

To find the value of the unlevered firm,

$V_L = V_U + PV(interest\ tax\ shields)$

$119.403\text{m} = V_U + 8.955m$

$V_U = 110.448\text{m}$

It makes sense that $V_U < V_L$ since D, and therefore IntExp, and therefore interest tax shield per year are all zero:

$interest\ tax\ shield\ per\ year = IntExp \times t_c = D \times r_d \times t_c$

Note that the value of the firm is higher when there's debt ($V_L \; compared \; to \; V_U$). This shows how leverage can add value due to its tax effect.

But does this mean that firms should be financed with 99% debt and 1% equity?

No, since this will lead to very large costs of financial distress which we'll discuss later.

Firm and Project Valuation

The main problems to avoid are:

- double-counting the interest tax shield in both the discount rate and the cash flow. It should only be included in one of them. Double-counting will lead to valuations that are too high.
- not including the tax shield at all, in the discount rate or in the cash flow. This will lead to valuations that are too low.

Theory Examples: Valuation

Question 1: A project's CFFA is calculated including the interest expense. It is discounted using the after-tax WACC. Is this correct?

Answer: No, this will double-count the interest tax shield. Interest is included in the cash flow (CFFA with interest expense), so the tax shield per year will be $\boldsymbol{IntExp}.\boldsymbol{t_c}$.

The discount rate is the after-tax WACC which also accounts for the tax shield since it reduced the cost of debt by $(1 - t_c)$. So the tax shield will be included in the cash flow and the discount rate which is wrong.

Question 2: A project's CFFA is calculated without any debt, that is, interest expense is excluded. It is discounted using the after-tax WACC. Is this correct?

Answer: Yes, this is the textbook method. The tax shield is only included in the discount rate. So long as the project has the same risk as the firm, and the firm keeps to its debt to equity ratio, then the project will be correctly valued.

Question 3: A project's CFFA is calculated assuming there is no debt, that is, interest expense is excluded. It is discounted using the pre-tax WACC. Is this correct?

Answer: No, tax shields are not included at all. This calculation will give the unlevered value of the project.

Question 4: A project's CFFA is calculated with a non-zero interest expense. It is discounted using the pre-tax WACC. Is this correct?

Answer: Yes, this is the harder method. The tax shield is included in the cash flow, and not the discount rate so this will give the correct, levered value of the project.

Another difficulty is that the WACC assumes a constant debt-to-equity ratio. This means that when the value of the firm changes, for example, after a dividend is paid, then the amount of debt and also the interest tax shield needs to be re-calculated. This can be very laborious.

The Costs of Financial Distress

While debt brings the benefits of lower tax payments in the form of interest tax shields, it also brings the costs of financial distress.

For example, take a car dealing business with a very high level of leverage. Say it has a debt-to-assets ratio of 10 to 1 (10:1 or 1,000%).

The car dealing firm will have high a interest expense which is a fixed cost. If the firm has a bad year with low sales, it will make a loss larger than its small amount of book equity, leading to bankruptcy.

The danger of bankruptcy will make the firm's employees afraid of losing their job when the firm goes bankrupt, so they will leave.

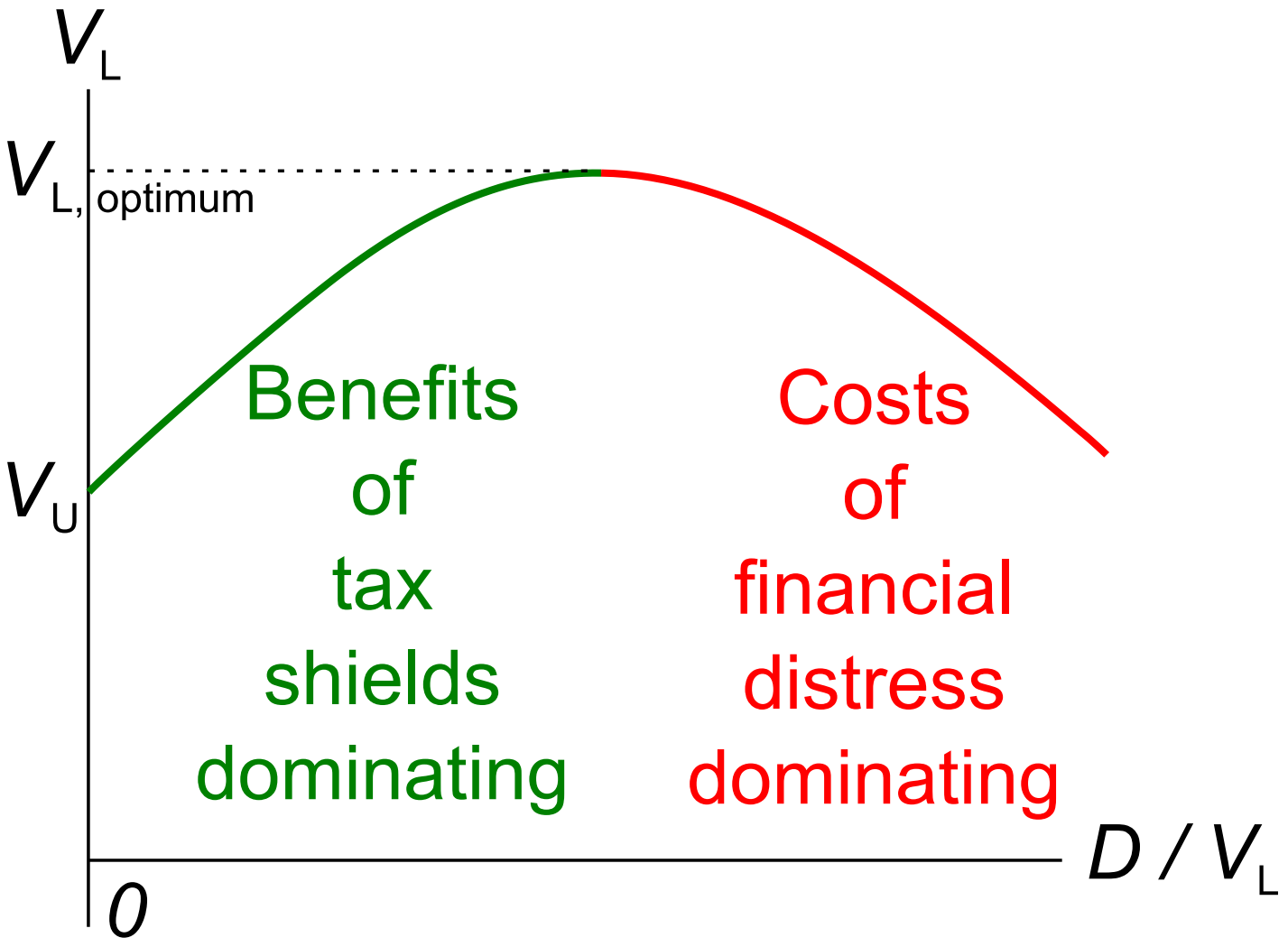
Customers will be wary of buying cars since the warranty will be worthless if the firm goes bankrupt.

Suppliers will also be wary of selling cars to the dealer on credit because if the dealer goes bankrupt then they may not be able to pay back the loan. So the supplier will demand cash on delivery and will not extend more generous credit terms.

All of these are examples of the real business costs of financial distress which become worse as the proportion of debt grows.

The costs of financial distress moderate the benefits of tax shields. Firms try to achieve a gearing (or leverage) ratio that

balances these costs and benefits. This is $V_{L,optimum}$ in the graph below.



Miller and Modigliani (M&M) - Capital Structure Irrelevance

In the 1950's, Miller and Modigliani, both Nobel Prize winners, described the conditions under which a firm's capital structure is irrelevant.

Their key argument was **'home-made' leverage.** The idea is that a firm's shareholders can borrow themselves, therefore they can create their own tax-shields. This means that a firm's capital structure is irrelevant.

The concepts are best illustrated in an example. This one is very interesting since it is a common misperception.

Question:

A firm is all-equity financed and has a value of V_U.

Management is thinking of increasing the proportion of debt in the firm's capital structure by selling debt (borrowing) and buying back shares (share repurchase). The assets of the firm will not be changed.

Management's argument for increasing the proportion of debt in the firm's capital structure is:

1. The firm's WACC is simply a weighted average of the cost of equity and the cost of debt.

$$WACC_{after-tax} = r_d.(1 - t_c).\frac{D}{V} + r_e.\frac{E}{V}$$

2. The cost of debt is always less than the cost of equity.
   $r_d < r_e$
3. Therefore by increasing the proportion of debt in the firm's capital structure, its WACC will be lower and so the levered value of the firm (V_L) will be higher.

Describe the **flaw** in the reasoning.

Consider the statements in a perfect world **with** and **without taxes**. The 'perfect world' has no transaction costs, information asymmetries or any other market frictions and the firm's shareholders can borrow at the same rate as the firm (r_d).

Answer: Management's first two statements are correct, but the third is incorrect.

Consider a 'perfect world' with no taxes ($t_c = 0$).

In this case there are at least three lines of thinking as to why the value of the firm (V_L) shouldn't change when equity is replaced with debt.

1. Interest tax shields would be zero **when the tax rate is zero**. Therefore the value of the firm must be the same since $V_L = V_U + PV(interest\ tax\ shields)$
2. The WACC is the required return on the firm's assets. Since the firm's assets are unchanged, the WACC should be unchanged. The risk of the firm's assets is still the same.
3. As the proportion of debt increases (higher leverage, so $\uparrow \frac{D}{V}$), the risk of equity increases and so does its required return

($\uparrow r_e$). This balances out the higher weight in the cheaper cost of debt in such a way that the WACC doesn't change.

Consider a 'perfect world' *with* taxes.

With taxes, it is logical that having more leverage ($\uparrow \frac{D}{V}$) will increase the value of the firm since interest tax shields will increase. But since investors can borrow themselves, they can use 'home-made' leverage to create their own tax shields. This means that management's efforts to increase leverage is pointless since investors can do it themselves. Thus a firm's capital structure is irrelevant, at least in this perfect world. This was the important insight provided by Miller and Modigliani.

