



SAMPLE FINAL EXAMINATIONS 2011 Version 3

Unit: ACCG253: Financial Management

Date: Tuesday 15 November, 2011 at 1:20pm

Time Allowed: 2 hours plus 10 minutes reading time.

Total Number of Questions: 10 Multiple Choice Questions plus 10 full response questions.

Instructions:

- PART A (30 marks):** There are 10 multiple choice questions. Answers to these must be recorded on a red-coloured General Purpose Answer Sheet which will be marked by a computer. Please make sure your name is on this sheet.
- PART B (90 marks):** There are 10 questions. Attempt all questions. Show all workings. Write answers in the spaces provided. Illegible handwriting risks loss of marks.

Materials Permitted:

- No dictionaries are permitted.
- A non-programmable calculator (no text retrieval capacity) is permitted.
- Financial calculators may be used.
- Mobile telephones must be turned off and left at the front of the room.

Question:	A	1	2	3	4	5	6	7	8	9	10	Total
Out of:	30	6	6	6	12	17	11	6	8	8	10	120
Mark:												

PART A (30 Marks)

There are **TEN (10)** multiple choice questions. Please select the most correct answer for each question. Each question in Part A is worth **3** marks. Record your answers on the computer readable answer sheet.

Question 1: Government bonds currently have a return of 5%. A stock has an expected return of 6% and the market return is 7%. What is the beta of the stock?

- (a) -0.5
- (b) 0
- (c) 0.5**
- (d) 1
- (e) 7

Question 2: The equations for Net Income (NI) and Cash Flow From Assets (CFFA) per year are:

$$NI = (Rev - COGS - Depr - IntExp) \cdot (1 - t_c)$$

$$CFFA = NI + Depr - CapEx - \uparrow NWC + IntExp$$

What is a formula for yearly *IntExp* (interest expense)? Note that *D* is the value of debt which is constant through time, and r_D is the cost of debt (yield on debt).

- (a) $D \times (1 + r_D)$
- (b) $D / (1 + r_D)$
- (c) $D \times r_D$**
- (d) D / r_D
- (e) $NI \times r_D$

Question 3: A manufacturing company is considering a new project in the **more risky** services industry. The cash flows from assets (CFFA) are estimated for the new project, with interest expense excluded from the calculations. To get the levered value of the project, what should these unlevered cash flows be discounted by?

Assume that the manufacturing firm has a target debt-to-assets ratio that it sticks to.

- (a) The manufacturing firm's before-tax WACC.
- (b) The manufacturing firm's after-tax WACC.
- (c) A services firm's before-tax WACC, assuming that the services firm has the same debt-to-assets ratio as the manufacturing firm.
- (d) A services firm's after-tax WACC, assuming that the services firm has the same debt-to-assets ratio as the manufacturing firm.**
- (e) The services firm's levered cost of equity.

Question 4: Which statement is the most correct?

(a) The more debt a firm has, the higher its tax shields. Therefore firms should seek to have as much debt and as little equity as possible.

(b) Debt makes a firm's equity more risky, therefore the higher the amount of debt, the higher the cost of equity.

(c) The before-tax cost of debt is less than the before-tax cost of equity. Therefore debt is a cheaper form of financing than equity so companies should try to finance their projects with debt only.

(d) The less debt, the lower the chance of bankruptcy. Therefore firms should try to pay off all of their debt so that they are financed by equity only.

(e) None of the above.

Question 5: Due to floods overseas, there is a cut in the supply of the mineral iron ore and its price increases dramatically. An Australian iron ore mining company therefore expects a large but temporary increase in its profit and cash flows. The mining company does not have any positive NPV projects to begin, so what should it do? Select the **most correct** answer.

- (a) Pay out the excess cash by increasing the regular dividend, and cutting it later.
- (b) Pay out a special dividend.
- (c) Conduct an on or off-market share repurchase.
- (d) Conduct a share dividend.
- (e) Either b or c.**

Question 6: Stocks A has a beta of 0.5 and stock B has a beta of 1. Which statement is **incorrect**?

- (a) Stock A has less systematic risk than stock B, so stock A's return should be less than stock B's.
- (b) Stock B has the same systematic risk as the market, so its return should be the same as the market's.
- (c) Stock B has the same beta as the market, so it also has the same total risk as the market.**
- (d) If stock A and B were combined in a portfolio with weights of 50% each, the beta of the portfolio would be 0.75.
- (e) Stocks A and B have more systematic risk than the risk free security (government bonds) so their return should be greater than the risk free rate.

The next 2 questions refer to the following table:

Portfolio Details					
Stock	Expected return	Standard deviation	Covariance	Beta	Dollars invested
A	0.2	0.4	0.12	0.5	40
B	0.3	0.8		1.5	80

Question 7: What is the **beta** of the above portfolio?

- (a) 0.75
- (b) 0.833333333
- (c) 1
- (d) 1.166666667**
- (e) 1.4

Question 8: What is the **standard deviation** (not variance) of the above portfolio? *(Remember to use the formula sheet at the back of this exam!)*

- (a) 0.5497
- (b) 0.5651
- (c) 0.5963**
- (d) 0.6156
- (e) 0.8165

Question 9: A firm's WACC **before** tax could increase due to:

- (a) the firm issuing more debt and using the proceeds to repurchase stock.
- (b) the firm issuing more equity and using the proceeds to pay off debt holders.
- (c) the firm's industry becoming more risky, for example if it was a mining company and commodities prices became more volatile.**
- (d) the firm's industry becoming less risky, for example if it was a child care centre and the government announced higher subsidies for parents using child care centres.
- (e) none of the above.

Question 10: A company has 50 million shares outstanding. The market price of one share is currently \$6. The risk-free rate is 5% and the market return is 10%. Market analysts estimated that the company's stock has a beta of 1.5.

The company has 1 million preferred stock which have a face (or par) value of \$100 and pay a constant dividend of 10% of par. They currently trade for \$80 each.

The company's debentures are publicly traded and their market price is equal to 90% of its face value. The debentures have a total face value of \$60,000,000 and the current yield to maturity of corporate debentures is 10% per annum. The corporate tax rate is 30%.

What is the company's after-tax weighted average cost of capital (WACC) under the classical tax system?

- (a) 11.12%
- (b) 11.75%
- (c) 11.82%**
- (d) 13.78%
- (e) 20.84%

PART B (90 Marks)

Answer each of the following questions in the space provided below the questions. Mark allocations are noted next to each question.

Question 1:

You own an apartment that you bought 4 years ago for \$320,000.

You currently rent it out for **\$2,800** per month. As is normal, rent is paid in advance (at the start of the month).

The current tenant will move out soon, and you are considering renovating the apartment when she moves out.

If you decide to renovate, it will cost **\$18,000** now to pay a builder who will take one month to complete the work. The apartment can't be rented while he is working. Once the work is finished, the apartment can be rented for **\$3,000** per month since it will be much nicer. A tenant should be available to move in straight away after the work is finished. Assume that the higher rent will last forever.

If you decide not to renovate, you can rent the apartment straight away after the current tenant moves out for the same monthly rent. Assume that the rent will also last forever.

The appropriate discount rate is **1%** per month, given as an effective monthly rate.

Question 1b (4 marks): What is the NPV of renovating the apartment?

$$\begin{aligned}
 \text{NPV} &= -\text{building cost now} - \text{month of lost rent now} + \text{PV of extra rent as a perpetuity.} \\
 &= -18k - 2.8k + \frac{3k - 2.8k}{0.1} \\
 &= -0.8k = -\$800
 \end{aligned}$$

Question 1b (1 mark):

Will the apartment ever pay itself back, using a pay-back period approach?

Circle: Yes or No

Question 1c (1 mark):

Circle whether the IRR of renovating the apartments will be:

more than, equal to, or less than the discount rate?

Question 2a (2 marks): In the context of capital budgeting, what is a 'sunk cost' and how is it dealt with when evaluating the NPV of a project?

Sunk costs are unrecoverable. They are ignored from NPV calculations since going ahead with the project (or not) will not lead us to pay that cost again or retrieve it either. It is gone, it is sunk.

Question 2b (4 marks): Your friend argues that opportunity costs should not be included in capital budgeting. His reasoning is as follows: depreciation expense is added back to cash flow from assets because it is a fictitious cash flow, it is made up by accountants. No-one actually pays the depreciation cost. Because no-one actually pays opportunity cost it should also not be included in capital budgeting when calculating cash flow from assets. Do you agree or disagree? Justify your answer.

The friend is wrong. Opp. costs do result in cash flows when the project is not undertaken, but when the project goes ahead then there is no cashflow.

For example, renting an office that you own is an opp. cost of doing your business project in the office. If you didn't do the business project, you wouldn't need the office & therefore you would receive the rent as a cashflow.

Question 3: A stock is priced at \$10 and will pay its annual dividend of \$1.50 in one year. The beta of the stock is 2, the risk free rate is 5% and the market return is 8%.

Question 3a (3 marks): What is the cost of equity using the CAPM (same as the SML)?

$$\begin{aligned}r_e &= r_f + \beta_e(r_m - r_f) \\&= 0.05 + 2(0.08 - 0.05) \\&= 0.11\end{aligned}$$

Question 3b (3 marks): Assuming that the CAPM gives the correct cost of equity, what must be the growth rate of dividends in the dividend discount model (DDM)?

$$P_{\text{share}} = \frac{\text{div}_1}{r_e - g}$$

$$10 = \frac{1.50}{0.11 - g}$$

$$0.11 - g = \frac{1.5}{10}$$

$$0.11 - \frac{1.5}{10} = g$$

$$g = -0.04$$

Question 4a (9 marks): Using the CAPM, give **three** reasons why the share price of a stock might fall. Explain your answer. (Hint: specify 3 things and whether they must increase or decrease for the stock price to fall).

$$r_e = r_f + B_e (r_m - r_f)$$

Since r_e is the discount rate of the cash flows to equity holders, if $r_e \uparrow$, then $P_{\text{share}} \downarrow$. r_e will \uparrow when:

$$\begin{array}{l} r_f \uparrow \\ B_e \uparrow \left(\text{Note } B_e = \frac{\text{cov}(r_e, r_m)}{\text{var}(r_m)} \right) \\ r_m \uparrow \end{array}$$

Question 4b (3 marks) What are the advantages and disadvantages of using the CAPM to estimate the cost of equity? List 3 points in total.

- (+) Makes an explicit compensation for systematic risk by incorporating beta into the equation. ($r_e = r_f + B_e (r_m - r_f)$)
- (+) Makes no assumptions about the level or growth rate of dividends, unlike the DDM.
- (-) Relies on estimates of r_f , r_m & B_e which can be difficult and are always changing.
- (+) Ignores idiosyncratic risk which is not relevant since it can be diversified away.

Question 5: Stocks, bonds, real estate and other assets have a certain amount of idiosyncratic and systematic risk. Answer the following:

Question 5a (4 marks): One of your friends is considering making a long-term investment of his life savings in the stock market. He really likes buying DVD's and goes to JB Hi-Fi all the time. He wants to invest in businesses he's familiar with. He mentions to you that he will invest all of his money into JB Hi-Fi shares. What advice would you give him?

Assume that JB Hi-Fi has a beta of 1, and that the CAPM holds and markets are efficient. In your answer, make explicit reference to **idiosyncratic** and **systematic** risk.

The friend should invest in a wider range of stocks to achieve diversification which will eliminate idiosyncratic risk. Systematic risk will still be present, but that risk is compensated by a higher return which is good.

Question 5b (12 marks): You have a small family and so do your three friends who live on your street. You each have the same amount of wealth and the same incomes.

Your friend **Alfred** used his wealth to buy the land where he lives. After buying the land outright, he hasn't been able to afford to build a house yet so he lives with his family in a tent on that land and doesn't have a mortgage.

Your friend **Betty** invested all of her money in government bonds and rents a modest house. She doesn't have a mortgage.

Your friend **Carl** bought the land where he lives and built a big mansion with a swimming pool, by borrowing a large amount using a mortgage.

You didn't take out a mortgage, but rent a modest house. You invested equal amounts in a very small apartment that you rent out, a portfolio of stocks and a portfolio of corporate bonds.

Rank each person's total, systematic and idiosyncratic risks from least to most risky. Write 1,2,3,4 in each box where 1 indicates the least risky. If you think that two persons' risks are the same, write the same number in their boxes:

	Total risk	Systematic risk	Idiosyncratic risk
Alfred	3	2	3
Betty	1	1	1
Carl	4	4	4
You	2	2	2

same syst. risk as you probably, but less divers.
Govt bonds have no risk at all.
Leverage increases all risks.
Diversified idios. risk the most.

Question 6: A firm has:

- A debt-to-equity ratio of 30%.
- A beta on equity of 1.5.
- Existing debt which yields 8%.
- A corporate tax rate of 30%.
- Also, the risk free rate is 5% and the market rate of return is 10%.

All rates are effective annual rates. Assume a **classical tax system** (no franking credits).

Question 6a (6 marks): Calculate the **cost of equity**, **pre-tax WACC** and **after-tax WACC**.

$$\textcircled{1} \quad r_e = r_f + \beta_e (r_m - r_f) \\ = 0.05 + 1.5 (0.1 - 0.05) = 0.125$$

$$\textcircled{3} \quad \text{WACC before tax} = \frac{E}{V} \cdot r_e + \frac{D}{V} \cdot r_D \\ = \frac{1}{1.3} \times 0.125 + \frac{0.3}{1.3} \times 0.08 \\ = 0.114615$$

$$\textcircled{2} \quad \frac{D}{E} = \frac{0.3}{1}$$

$$V = D + E \\ = 0.3 + 1 \\ = 1.3$$

$$\frac{D}{V} = \frac{0.3}{1.3}$$

$$\frac{E}{V} = \frac{1}{1.3}$$

$$\textcircled{4} \quad \text{WACC after tax} = \frac{E}{V} \cdot r_e + \frac{D}{V} \cdot r_D (1 - t_c) \\ = \frac{1}{1.3} \times 0.125 + \frac{0.3}{1.3} \times 0.08 (1 - 0.3) \\ = 0.109077$$

Question 6b (3 marks): If the firm decides to issue bonds and use the proceeds to repurchase stock, without buying or selling any assets, state which of the above 3 rates will increase, decrease or stay the same.

Since firm will have more leverage $\left(\frac{D}{E} \uparrow\right)$,

- $r_e \uparrow$ since equity will become more risky.
- WACC before tax will stay the same since assets are unchanged.
- WACC after tax will fall due to benefit of tax shields.

Question 6c (2 marks): Often, the cost of debt is assumed to stay the same when capital structure changes. In the above scenario, when the firm issues bonds and uses the proceeds to repurchase stock, what is likely to happen to the cost of debt? Will it increase, decrease or stay the same? Justify your answer.

The cost of debt, r_D , is likely to increase because with more debt, the chance of the firm going bankrupt is higher & debtholders may not get paid back in full. This makes debt more risky, so the return r_D will increase.

Question 7: Stocks A and B have the following returns:

Year	r_A	r_B
2007	0.2	0.4
2008	0.04	-0.2
2009	-0.1	-0.3
2010	0.18	0.5

Question 7a (1 marks): What is the average return of each stock?

$$\bar{r}_A = \frac{0.2 + 0.04 - 0.1 + 0.18}{4} = 0.08$$

$$\bar{r}_B = \frac{0.4 + -0.2 - 0.3 + 0.5}{4} = 0.1$$

Question 7b (3 marks): What is the standard deviation of returns for each stock?

$$\sigma_A^2 = \left((0.2 - 0.08)^2 + (0.04 - 0.08)^2 + (-0.1 - 0.08)^2 + (0.18 - 0.08)^2 \right) \div (4 - 1)$$

$$= 0.019467, \quad \sigma_A = 0.1395$$

$$\sigma_B^2 = 0.166667$$

$$\sigma_B = 0.408248$$

Question 7c (2 marks): What is the correlation and covariance between stock A and stock B's returns?

$$\sigma_{A,B} = \left((0.2 - 0.08)(0.4 - 0.1) + (0.04 - 0.08)(-0.2 - 0.1) + (-0.1 - 0.08)(-0.3 - 0.1) + (0.18 - 0.08)(0.5 - 0.1) \right) \div (4 - 1)$$

$$= 0.053333$$

$$\rho_{A,B} = \frac{\sigma_{A,B}}{\sigma_A \cdot \sigma_B} = \frac{0.053333}{0.1395 \times 0.408248} = 0.9363$$

Question 8

Question 8a (3 marks): Read the following trading strategies, and label what type of market efficiency is broken in each (strong, semi-strong or weak form market efficiency):

i) An advertisement for a computer trading program states that it can analyse past return and volatility trends to predict stock performance and select the best stocks to buy and sell, giving traders positive abnormal returns.

Breaks weak form mkt efficiency, since it is using past prices only. Stock prices are supposed to be a random

ii) Your friend who is an engineer working for an exploration mining company he tells you that he is willing to give you advance news of any new potential resource deposits found in the outback, before other shareholders find out. He says that if you buy the stock before the announcement is made, you can make positive abnormal returns. walk with an upward trend that is commensurate with the level of risk.

This breaks strong form mkt effie. Inside information is being used.

iii) Using Google's internet search engine you find that you can pick stocks that will fall, and sell them in advance. You simply search for the stock's name together with the key-word "trouble", and if there are more than 20,000 hits then that tells you to sell that stock. Using this strategy, you make positive abnormal returns.

This breaks semi-strong form effie. The strategy is using publically available information.

Question 8b (5 marks): List the three forms of market efficiency in the order that they are likely to hold in the real world, and **justify** your choice.

1. Most likely to hold: weak form. Because it is so easy for a computer program to find and exploit any simple strategies based on past prices. Past prices are readily available & easy to access.

2. Less likely to hold: semi-strong. Because although public information is public, it still takes time to evaluate, & is costly to interpret. Therefore there is likely to be some opportunities to make positive excess returns.

3. Least likely to hold:

Strong form. Because if this form of efficiency held, people would be breaking the law & conducting insider trading.

Question 9: An investor has \$5 million to invest in shares A and B. The investor seeks a portfolio beta of 0.5.

Share A has a beta of 1.2.

Share B has a beta of 0.2.

The risk free rate is 5% and the market return is 10% pa.

Question 9a (5 marks): How much money should the investor invest in share A and share B respectively?

$$\beta_p = x_1 \cdot \beta_1 + x_2 \cdot \beta_2$$

$$0.5 = x_A \times 1.2 + x_B \times 0.2$$

$$0.5 = 1.2x_A + 0.2(1 - x_A)$$

$$0.5 = 1.2x_A + 0.2 - 0.2x_A$$

$$0.3 = x_A,$$

$$x_B = 1 - 0.3 = 0.7.$$

$$V_A = x_A \cdot V_P$$

$$= 0.3 \times \$5m = \$1.5m$$

$$V_B = x_B \cdot V_P$$

$$= 0.7 \times \$5m = \$3.5m$$

Question 9b (3 marks): Stock A has total standard deviation of 40% and stock B has a total standard deviation of 50%. Find portfolio total standard deviation and portfolio systematic standard deviation.

$$\sigma_m = 0.3$$

$$\sigma_{1,2} = 0.0216$$

$$\sigma_p^2 = x_A^2 \cdot \sigma_A^2 + x_B^2 \cdot \sigma_B^2 + 2x_A x_B \sigma_{A,B}$$

$$= 0.3^2 \times 0.4^2 + 0.7^2 \times 0.5^2 + 2 \times 0.3 \times 0.7 \times 0.0216$$

$$= 0.141436 \leftarrow \text{This is total variance.}$$

$$\sigma_p = \sqrt{0.141436} = 0.37608 \leftarrow \text{total standard dev.}$$

$$\sigma_{p, \text{total}}^2 = \underbrace{\beta_p^2 \cdot \sigma_m^2}_{\text{sys.}} + \sigma_{p, \text{E}}^2$$

$$\sigma_{p, \text{sys}}^2 = \beta_p^2 \cdot \sigma_m^2 = 0.5^2 \times 0.3^2 = 0.0225^{15}$$

$$\sigma_{p, \text{sys}} = \sqrt{0.0225} = 0.15$$

Question 10 (10 marks):

Project Data	
Project life	2 yrs
Initial investment in factory upgrades	\$3m
Unit sales per year	2m
Sale price per unit	\$16
Variable cost per unit	\$5
Fixed costs per year, paid at the end of each year	\$2m
Tax rate	0.3

Note 1: The factory upgrades can be depreciated straight line to zero over the life of the project. The upgrades are worthless at the end of the project's life.

Side benefit { **Note 2:** The factory upgrades can be used to help the firm's other projects which is expected to add \$0.5m every year to the firm's revenues. The amount is before tax, and is paid at the end of each year over the life of the project.

Sunk cost { **Note 3:** The new machinery that will be installed in the factory upgrades will make a lot of noise, so the local council tried to block the development application. To placate the council and local residents, the firm agreed to develop a nearby swamp into a nice park, which has been completed and cost \$0.8m.

Find the three Cash Flow From Assets (CFFA) at times $t=0, 1, 2$. You DO NOT need to find NPV.

$$\text{Depr} = \frac{\$3\text{m}}{2} = \$1.5\text{m}$$

$$\text{CFFA} = \text{NI} + \text{Depr} - \text{CapEx} - \text{TNWC} + \text{IntExp}$$

$$\begin{aligned} \text{NI} &= (Q(P - VC) - FC - \text{Depr} - \text{IntExp})(1 - t_c) \\ &= (2\text{m}(16 - 5) - 2\text{m} + \underset{\substack{\uparrow \\ \text{side} \\ \text{benefit}}}{0.5\text{m}} - 1.5\text{m} - 0)(1 - 0.3) \\ &= \$13.3\text{m} \end{aligned}$$

$$\text{CFFA}_0 = 0 + 0 - 3\text{m} - 0 + 0 = -\$3\text{m}$$

$$\text{CFFA}_1 = 13.3\text{m} + 1.5\text{m} - 0 - 0 + 0 = \$14.8\text{m}$$

$$\text{CFFA}_2 = 13.3\text{m} + 1.5\text{m} - 0 - 0 + 0 = \$14.8\text{m}$$

Spare Scribble Sheet

Formulas

$$PV(\text{single cash flow}) = V_0 = \frac{C_t}{(1 + r_{eff})^t}$$

$$PV(\text{annuity}) = V_0 = \frac{C_1}{r_{eff}} \left(1 - \frac{1}{(1 + r_{eff})^T} \right)$$

$$PV(\text{perpetuity}) = V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$r_{eff,annual} = (1 + r_{eff,monthly})^{12} - 1$$

$$r_{eff,monthly} = \frac{r_{APR,comp\ monthly}}{12}$$

$$Price_{bill} = V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365} \right)}$$

$$Price_{bond} = PV(\text{annuity of coupons}) + PV(\text{principal})$$

$$= \frac{C_1}{r_{eff}} \left(1 - \frac{1}{(1 + r_{eff})^T} \right) + \frac{Face}{(1 + r_{eff})^T}$$

$$CapEx = NFA_{now} - NFA_{before} + Depreciation$$

$$\uparrow NWC = (CA_{now} - CL_{now}) - (CA_{before} - CL_{before})$$

$$CFFA = NI + Depr - CapEx - \uparrow NWC + IntExp$$

$$CFFA = CF \text{ to equity holders} + CF \text{ to creditors}$$

$$r_{0-1} = \frac{p_1 - p_0}{p_0} = \frac{p_1}{p_0} - 1$$

$$\bar{r} = \frac{\sum_{i=1}^n (r_i)}{n} = \frac{r_1 + r_2 + \dots + r_n}{n}$$

$$var(r) = \sigma^2 = \frac{\sum_{i=1}^n [(r_i - \bar{r})^2]}{n - 1}$$

$$cov(r_1, r_2) = \sigma_{1,2} = \frac{\sum_{i=1}^n [(r_{1,i} - \bar{r}_1)(r_{2,i} - \bar{r}_2)]}{n - 1}$$

$$correl(r_1, r_2) = \rho_{1,2} = \frac{cov(r_1, r_2)}{sd(r_1).sd(r_2)} = \frac{\sigma_{1,2}}{\sigma_1.\sigma_2}$$

$$\bar{r} = \sum_{i=1}^n (p_i \cdot r_i) = p_1 \cdot r_1 + p_2 \cdot r_2 + \cdots + p_n \cdot r_n$$

$$var(r) = \sigma^2 = \sum_{i=1}^n [p_i (r_i - \bar{r})^2]$$

$$cov(r_1, r_2) = \sigma_{1,2} = \sum_{i=1}^n [p_i (r_{1,i} - \bar{r}_1)(r_{2,i} - \bar{r}_2)]$$

$$r_P = x_1 \cdot r_1 + x_2 \cdot r_2 + \cdots + x_n \cdot r_n = \sum_{i=1}^n (x_i \cdot r_i)$$

$$x_1 + x_2 + \cdots + x_n = 1$$

$$\sigma_P^2 = x_1^2 \cdot \sigma_1^2 + x_2^2 \cdot \sigma_2^2 + 2 \cdot x_1 \cdot x_2 \cdot \sigma_{1,2}$$

$$\sigma_{1,2} = \rho_{1,2} \cdot \sigma_1 \cdot \sigma_2$$

$$\mu_i = r_f + \beta_i (\mu_M - r_f)$$

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2} = \frac{cov(r_i, r_M)}{var(r_M)}$$

$$\sigma_{i,total}^2 = \beta_i^2 \cdot \sigma_M^2 + \sigma_{i,\varepsilon}^2$$

$$\beta_P = x_1 \beta_1 + x_2 \beta_2 + \cdots + x_n \beta_n$$

$$r_{WACC \text{ after tax}} = \frac{D}{V} \cdot r_D (1 - t_c) + \frac{E}{V} \cdot r_E$$

$$r_{WACC \text{ before tax}} = r_A = \frac{D}{V} \cdot r_D + \frac{E}{V} \cdot r_E$$

$$V = D + E$$