Financial Management
AFIN253

Lecture 2

Advanced Equity and Debt Valuation

Authors: Keith Woodward and Damian Bridge

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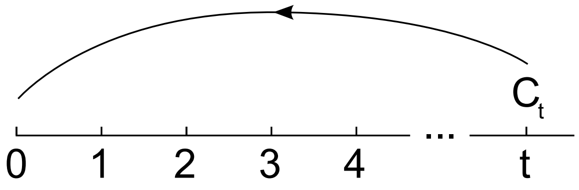
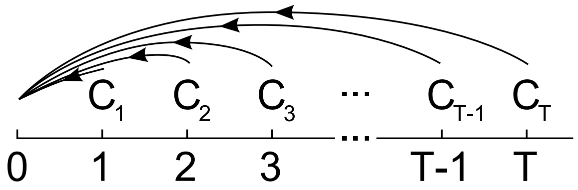
Revision: Present Value Formulas

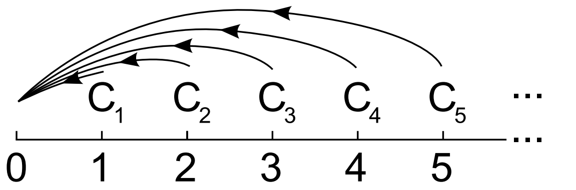
$$PV(single\ cash\ flow) = V_0 = \frac{C_t}{(1 + r_{eff})^t}$$



$$PV(annuity) = V_0 = \frac{C_1}{r_{eff}}\left(1 - \frac{1}{(1+r_{eff})^T}\right)$$



$$PV(perpetuity) = V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$



$$r_{eff,monthly} = r_{APR,comp\ monthly} \div 12$$

Asset Classes

The main investable asset classes are:

- Equity, also known as stocks and shares.
- Debt, which is usually divided into:
  - long term debt such as bonds and loans and
  - short term debt with a maturity of less than 1 year such as bank accepted bills (BAB's), certificates of deposit (CD's) and promissory notes (PN's).
- Real estate such as land, houses, apartments and buildings.

Derivatives can be considered another asset class, but they are quite different and are mostly used for hedging (reducing risk) and speculating (informed gambling) rather than investment.

Income, Capital and Total Returns

Returns on stocks, bonds, real estate, and any asset can be broken into two parts, the income return and the capital return.

The **income return** is the proportion of the asset's price that is paid out in cash per time period.

$$\boldsymbol{r_{income,0-1} = \frac{C_1}{P_0}}$$

Where C_1 is the cash flow at t=1 and P_0 is the price at t=0.

The cash flow income:

- from a stock is a dividend,
- from a bond is a coupon,
- from real estate is rent.

The **capital return** is the rate of increase in the asset's price per time period.

$$r_{capital,0-1} = \frac{P_1 - P_0}{P_0}$$

Total return is the sum of the income return and the capital return.

$$\begin{aligned} r_{total,0-1} &= r_{capital,0-1} + r_{income,0-1} \\ &= \frac{P_1 - P_0}{P_0} + \frac{C_1}{P_0} \\ &= \frac{P_1 - P_0 + C_1}{P_0} \end{aligned}$$

Calculation Example: Components of Returns

Question: A stock was bought for \$10 at $t=0$.

At $t=1$ the stock paid a dividend of \$1 and immediately afterwards the price of the stock was \$9.50.

At $t=2$ the stock paid no dividend and its price was \$12.

All time periods are measured in years.

Find the total, dividend and capital returns of the stock over the first and second years.

Answer:

Over the first year (from t=0 to t=1):

$$r_{income,0-1} = \frac{C_1}{P_0} = \frac{1}{10} = 0.1 = 10\%$$

$$r_{capital,0-1} = \frac{P_1 - P_0}{P_0} = \frac{9.50 - 10}{10} = -0.05 = -5\%$$

$$r_{total,0-1} = r_{income,0-1} + r_{capital,0-1}$$

$$= \quad 0.1 \quad + \quad -0.05 \quad = 0.05 = 5\%$$

Over the second year (from t=1 to t=2):

$$r_{income,1-2} = \frac{C_2}{P_1} = \frac{0}{9.50} = 0 = 0\%$$

$$r_{capital,1-2} = \frac{P_2 - P_1}{P_1} = \frac{12 - 9.50}{9.50} = 0.263157895 = 26.32\%$$

$$r_{total,1-2} = r_{income,1-2} + r_{capital,1-2}$$

$$= \quad 0 \quad + \; 0.263157895$$

$$= 0.263157895 = 26.32\%$$

Note that all of these returns are effective annual rates.

Asset Valuation

Assets can be valued using two main methods:

- Discounted Cash Flow (DCF) valuation
- Multiples valuation

While any assets including debt, equity and real estate can be valued using these techniques, we will focus on the valuation of equity.

Equity: Discounted Cash Flow Valuation

Discounted cash flow (DCF) valuation involves finding the present value of the future cash flows of the stock.

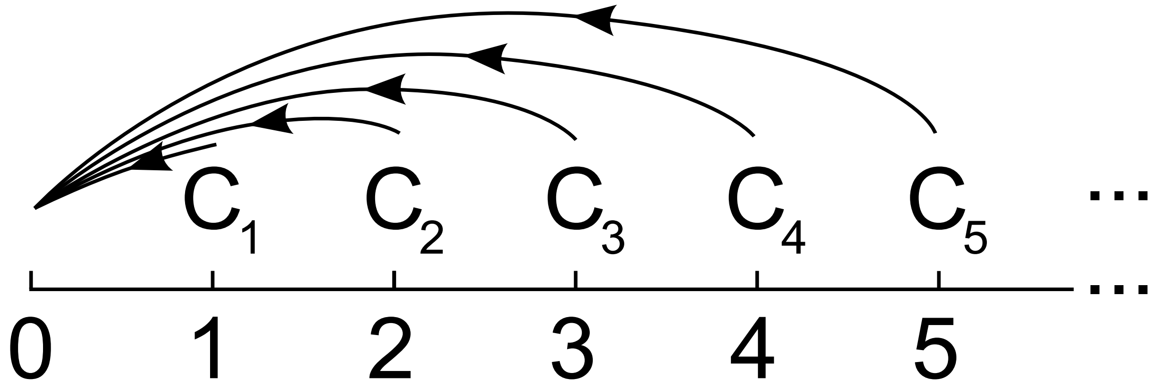
Commonly this involves using the 'perpetuity with growth' formula, also called the 'Gordon Growth Model' or the 'Dividend Discount Model' (DDM).

$$\boldsymbol{PV(perpetuity\ with\ growth) = V_0 = \frac{C_1}{r - g}}$$

C_1 = dividend cash flow received at $t = 1$. The dividend cash flows go on forever, but grow by g every period.

g = effective growth rate of the dividend over a single period.

$r =$ effective discount rate over a single period, also called the required return on equity.



The Dividend Discount Model (DDM)

The DDM (or perpetuity with growth formula), has some important assumptions that you need to be aware of:

$$\boldsymbol{V_0 = \frac{C_1}{r - g}}$$

- The first dividend or cash flow in the formula occurs at t=1, not at t=0.
- The constant dividend growth rate 'g' must occur in perpetuity, that is, forever.
- The growth in the dividend 'g' is also the capital growth rate of the stock, also known as price growth or the capital return ($\boldsymbol{r_{capital}}$).
- $\boldsymbol{r}$ is the total return of the stock ($\boldsymbol{r_{total}}$).

The Dividend Discount Model in Practice

A common mistake when applying the DDM is to use a growth rate that is too high. Remember that the dividend growth is also the capital return (proportional price increase) and it is perpetual.

Say there is a stock, 'Growth Corp', that has a high average historical dividend growth rate of 7% pa. An inexperienced analyst forecasts that dividends will continue to grow at this high rate forever.

Since the average stock in the economy grows by approximately the level of GDP growth, which is around 5% pa, then Growth Corp will outgrow the average stock. But since this occurs in perpetuity, Growth Corp will take over the world since it is always getting bigger than the average firm forever.

So there is an upper limit on the nominal dividend growth rate which is nominal GDP growth.

Equity: Multiples Valuation

Multiples valuation is the preferred method to value stocks amongst practitioners in industry. There are many different multiples that are used, some of the most common are:

- Share Price/Earnings (PE) ratio,
- Enterprise Value/EBITDA (EV/EBITDA) ratio
- Book value of equity/Market value of equity (Book to market) ratio
- Share Price/ Sales (Price to sales) ratio
- (Share Price/Earnings)/Earnings growth (PEG) ratio

We will focus on the 'Price to Earnings' or PE ratio.

Earnings Per Share (EPS) Calculation

Earnings per share is reported in company's financial reports. It is the total earnings of the firm divided by the total number of shares.

$$EPS = \frac{Total\ earnings}{number\ of\ shares}$$

Note that earnings are an American term for Net Income (NI) or Net Profit After Tax (NPAT).

PE Ratio Calculation

The PE ratio can be calculated in two different ways which give the same answer:

$$PE\ ratio = \frac{share\ price}{EPS}$$

If we multiply the top and bottom of the fraction by the total number of shares, then the denominator will be total earnings, and the numerator will be the market capitalisation of equity which is the price of buying all shares.

$$PE\ ratio = \frac{market\ capitalisation\ of\ equity}{total\ earnings}$$

Price-Earnings Ratio Valuation

Assume that stock XYZ's market capitalisation of equity is being valued using price-earnings multiples.

- Make a list of similar firms from the same industry as XYZ with the same levels of risk and leverage (ratio of debt to assets).
- Calculate each similar firm's PE ratio by dividing its current share price by its earnings per share last year (historical EPS). Calculate the average of all of the similar firms' PE ratios. If any firms had negative EPS or EPS close to zero, then their PE ratios will be negative or extremely large so they should be excluded from the average.
- Multiply XYZ's EPS last year by the average PE ratio of similar firms. This will give the share price of XYZ. Multiplying by the number of shares gives its market capitalisation of equity.

Backward versus Forward Looking PE Ratios

In the above steps we valued our firm using 'backward loooking' PE ratios since we used historical EPS. This gives PE ratios which are accurate but stale since they reflect the past, not the future which is what we're interested in.

Another way of doing PE ratio valuation is to use 'forward-looking' PE ratios by using next year's expected EPS which are more useful, but less accurate because. They are less accurate because next year's EPS is unknown so they are usually based on analysts' forecasts which can vary widely.

Reconciling DCF and PE-Ratio Valuation

Discounted Cash Flow (DCF) valuation and Price/Earnings (PE) ratio valuation can be seen as two sides of the same coin if we make a few assumptions.

Assume that a firm pays out all of its earnings as dividends. In this case, there will be no growth in earnings, dividends or stock price since there is no re-investment back into the firm to buy new assets and make higher earnings.

Using DCF valuation we will apply the Dividend Discount Model (DDM) as follows:

$$P_0 = \frac{C_1}{r - g}$$

But the dividend cash flow C_1 will be equal to earnings (earnings_1), and g will equal zero since there is no growth in earnings. Therefore:

$$P_0 = \frac{\text{earnings}_1}{r - 0}$$

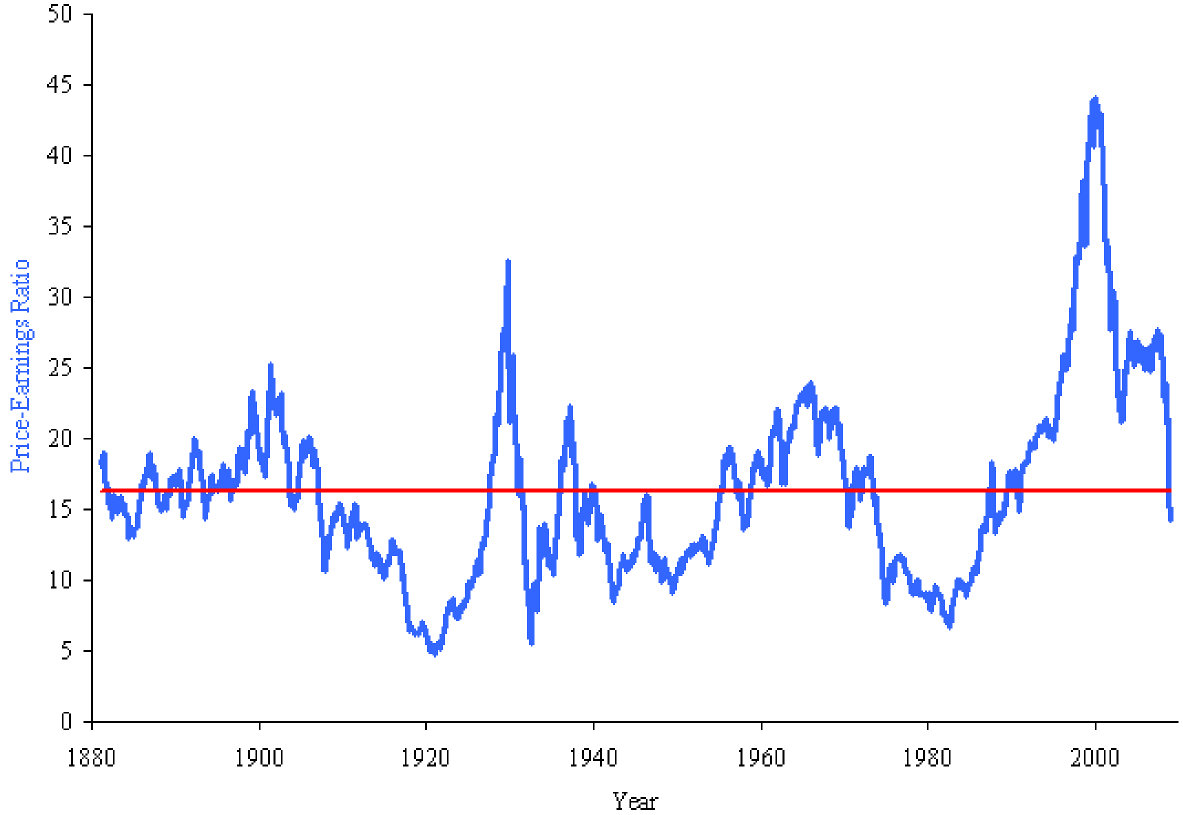
$$P_0 = \frac{\text{earnings}_1}{r}$$

Re-arranging,

$$\frac{P_0}{earnings_1} = \frac{1}{r}$$

The left hand side of this equation is actually the forward-looking PE ratio formula, it's the ratio the current share price (at t=0) to next year's earnings (at t=1).

$$\frac{P_0}{earnings_1} = \frac{1}{r}$$



So the PE ratio can be seen as (1/r), the inverse of the required return on equity.

This makes sense because PE ratios are generally around 15, and the real total required return on stocks is likely to be around 7%, and (1/0.07) is approximately 15. This can be seen in Robert Shiller's historical Price-Earnings graph above.

Overview of DCF and Multiples Approaches

Discounted Cash Flow (DCF) valuation

- This is the Net Present Value (NPV) of cash flows.
- Preferred when future cash flows are predictable and the required return can be easily calculated.
- Debt securities, real estate with stable rental income and stable low-growth stocks are suitable for DCF valuation.
- More of an absolute or intrinsic valuation technique, as opposed to a relative valuation technique.
- Solid mathematical foundation and theory, favoured by academics.

Multiples valuation

- Relative valuation technique. Prices assets using the prices of other, similar assets.
- Many different types of multiples can be used.
- Simple, intuitive, based on real-world prices.
- Preferred when future cash flows are unpredictable, and when there are many similar assets that are frequently traded at observable prices, such as stocks trading on a stock exchange.
- Real estate and stocks are suitable for multiples valuation.

Types of Debt Securities

Short term debt securitites

Bills, commonly Bank Accepted Bills (BAB's)

Certificates of Deposit (CD's)

Promissory notes (PN's)

Long term debt securities

Bonds. The Reserve Bank defines bonds as having a maturity of more than one year from the date of issue.

Loans are not debt securities

Loans are a type of debt but they are not securities. This is because loans are not fungible which means that one loan can't be substituted for another. Non-fungible assets are illiquid.

For example, a firm wants to borrow $100 million. If it is borrowed from the bank as a loan, there is one loan contract between the firm and the bank. If it is raised using bonds, each investor will have an identical bond contract, and they can easily sell their bond to another investor if they want.

The bonds are all the same, so they are fungible.

Bills, shares, and exchange traded derivatives like options and futures are also fungible. Real estate is not fungible.

Short term debt securities

Usually have a maturity of less than 1 year when issued.

Yields are quoted as simple annual rates, which are very different to compound rates such as effective rates and APR's.

Most short term debt securities are 'discount securities', which means that they do not pay coupons. The price of a discount security will always be less than the face value.

$$Price_{bill} = V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

Where F_t is the face value, r_{simple} is the simple annual interest rate and t is days until maturity.

Calculation Example: BAB's

Question: A company issues a bill which is accepted (guaranteed) by a bank. The BAB will mature in 90 days, has a face value of $1 million and an interest rate of 7% pa. What is the price of the bill?

Answer:

$$V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

$$= \frac{1{,}000{,}000}{\left(1 + 0.07 \times \frac{90}{365}\right)}$$

$$= 983{,}032.5882$$

Long term debt securities: bonds

Usually have an original maturity of more than 1 year.

Yields are quoted as APR's, compounding at the same frequency as the coupons are paid. Other names for yield include discount rate, internal rate of return, interest rate or required return.

Most bonds pay coupons. Coupon payments are calculated as face value multiplied by the coupon rate. If the coupons are paid semi-annually, then half of the total coupon is paid every 6 months. Coupon rates are commonly confused with yields, but they are different. Coupon rates are just a convenient way to specify coupon payments.

Bond Pricing

The price of a bond is the present value of the coupons and the principal. The coupons are an annuity and the principal (or face value) is a single payment, therefore:

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

Care must be taken when choosing r_{eff} and T so that they are consistent with the time period between coupon payments (C_1). Remember that bond yields are given as APR's compounding at the same frequency as coupons are paid.

Bond Pricing Conventions

US and Australian bonds generally* pay semi-annual coupons. Therefore the yields are quoted as APR's compounding semi-annually.

European bonds pay annual coupons. Therefore the yields are quoted as APR's compounding annually, which is the same thing as an effective annual rate.

* Inflation linked bonds are a rare exception. They pay quarterly coupons since inflation statistics are released quarterly. Therefore the yield on an inflation linked bond is quoted as an APR compounding per quarter.

Calculation Example: Coupon bonds

Question: An Australian company issues a bond. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8%. Yields are currently 5% pa. What is the price of the bond?

Answer: Since it is an Australian bond, we assume that it pays semi-annual coupons as is customary.

Therefore each 6 month coupon will be:

$$C_{semi-annual} = Face\ value \times \frac{coupon\ rate}{2}$$

$$= 1{,}000 \times \frac{0.08}{2} = \$40$$

The number of time periods T must be consistent with the coupon payment frequency of 6 month periods, so

$$T = 3\text{ years } \times 2$$
$$= 6\text{ semi-annual periods}$$

The yield of 5% can be assumed to be an APR compounding every 6 months, the same frequency as the coupon payments. We need to find the effective 6 month rate to discount the 6-month coupons, so:

$$r_{eff,6month} = r_{APR,comp\ semi-annually} \div 2$$
$$= 0.05 \div 2 = 0.025$$

To find the bond price,

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{40}{0.025}\left(1 - \frac{1}{(1 + 0.025)^6}\right) + \frac{1{,}000}{(1 + 0.025)^6}$$

$$= 220.3250145 \quad + \quad 862.296866$$

$$= \$1{,}082.62$$

Bond Yields and Coupon Rates

Here are three similar bonds which differ only in their coupon rate.

Bond	Face value	Maturity	Coupon rate	Current yield	Price	Exact price*	**Bond type**
1	$100	3 yrs	0%	5%	< $100	$86.23	**Discount**
2	$100	3 yrs	5%	5%	= $100	$100.00	**Par**
3	$100	3 yrs	10%	5%	> $100	$113.77	**Premium**

* Coupons are assumed to be paid semi-annually.

- Bonds issued at 'par' will have:
  - A price equal to face value
  - A yield equal to the coupon rate.
- Zero coupon bonds are discount bonds.

Calculation Example: Bonds issued at par

Question: An Australian company issues a bond at **par**. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8%. What is the price of the bond?

Answer: This is a trick question, no calculations are required. Since the bond was issued at par, the price must be equal to the face value. Therefore the price is $1,000. Current yields in the bond market must also be equal to 8% pa, the same as the bond's coupon rate.

Calculation Example: Zero coupon bonds

Question:

An Australian company issues a zero coupon bond. The bond will mature in 3 years and has a face value of $1,000. If the current price of the bond is $700, what is the current yield on the bond?

Answer:

Zero coupon bonds pay no coupons. Therefore the price of the bond is just the present value of the principal.

To find current yields we need to solve for the discount rate:

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= 0 + \frac{Face}{\left(1 + r_{eff}\right)^{T}}$$

$$700 = \frac{1{,}000}{\left(1 + r_{eff,6mth}\right)^{6}}$$

$$700 \times \left(1 + r_{eff,6mth}\right)^{6} = 1{,}000$$

$$\left(1 + r_{eff,6mth}\right)^{6} = \frac{1{,}000}{700}$$

$$1 + r_{eff,6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}}$$

$$r_{eff,6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}} - 1$$

$$= 0.061248265$$

We need to convert this rate to an APR compounding every 6 months since that is how bond yields are quoted in Australia.

$$r_{APR,comp\ per\ 6mths} = r_{eff,6mth} \times 2$$

$$= 0.061248265 \times 2$$

$$= 0.12249653 = 12.249653\%$$

For the exam, note that you will not be asked to find the yield on coupon-paying bonds since that requires trial-and-error or a computer. But you may be asked to find the yield on a zero coupon bond like we did here.

Term Structure of Interest Rates

Long term interest rates are based on expectations of future short term interest rates.

We will discuss spot and forward rates, yield curves, and then two important theories of interest rates:

- Expectations hypothesis
- Liquidity premium theory

Spot and Forward Interest Rates

Spot rate: An interest rate measured from now until a future time. For example, a 3-year zero-coupon bond with a yield of 8% pa has a 3-year pa spot rate of $r_{0-3,yearly} = 0.08$ pa. Note that spot rates can be from now until ***any*** future time.

Forward rate: An interest rate measured from a future time until a more distant future time. For example, if a company promised, **one year from now,** to issue a 3-year zero-coupon bond with a yield of 8% pa, then the forward rate from years 1 to 4 would be $r_{1-4,yearly} = 0.08$. Forward rates are sometimes written with an 'f' rather than 'r'.

Spot and forward rates can be quoted as APR's or effective rates.

Yield Curves

Yield curves show the behaviour of short and long-term interest rates and can give an indication of expected future interest rates.

Yield curves can be flat, normal, inverse, or some combination. The x-axis of a yield curve is the time to maturity of the bond, and the y-axis is the yield of the bond.

A **flat** yield curve is a straight horizontal line. Short and long term spot rates are equal, and yearly spot and forward rates are also equal. Other names for flat interest rates are 'constant', 'unchanging', or 'level' interest rates.

A **normal** yield curve is an upward sloping line or curve. Short term spot rates are less than long term spot rates. Yearly spot rates are less than yearly forward rates. Other names for normal yield-curves are 'upward sloping' or 'steep ' yield curves. These yield curves are 'normal' since yields usually exhibit this behaviour.

An **inverse** yield curve is a downward sloping line or curve. Short term spot rates are more than long term spot rates. Yearly spot rates are more than yearly forward rates. Other names for inverse yield-curves are 'downward sloping' or 'inverted ' yield curves.

Term Structure of Interest Rates: The Expectations Hypothesis

Expectations hypothesis is that long term spot rates (plus one) are the geometric average of the shorter term spot and forward rates (plus one) over the same time period. Mathematically:

$$r_{0-T} = \left((1 + r_{0-1})(1 + r_{1-2})(1 + r_{2-3}) \ldots (1 + r_{(T-1)-T})\right)^{\frac{1}{T}} - 1$$

or the more intuitive, memorable version:

$$(1 + r_{0-T})^T = (1 + r_{0-1})(1 + r_{1-2})(1 + r_{2-3}) \ldots (1 + r_{(T-1)-T})$$

Where T is the number of periods and all rates are effective rates over each period.

An Extension: Liquidity Premium Theory

The expectations hypothesis assumes that investors are indifferent between investing in a 10 year bond, or investing in a one year bond, then investing in another 1 year bond after the first is repaid, and so on for 10 years.

Most investors would prefer to lend lots of short term bonds rather than one big long one. The reason is that the long-term bond locks up the investor's cash and she loses the option to change her mind and do something else with the cash.

The liquidity premium theory suggests investors are only enticed to lend their cash out long-term if they are rewarded for doing so in the form of higher long-term rates compared to short term rates. This means that forward rates will be higher than the expected spot rates over the same time period.

For example, if the forward rate from years 1 to 2 is 8% now, then 1 year later the spot rate (from years 0 to 1) would tend to be less, say 7.5%.

This theory explains why the up-ward sloping yield curve is normal, since spot rates would tend to be less than forward rates.

Calculation Example: Term Structure of Interest Rates

Question: The following US Government Bond yields were quoted on 5/3/2012 (sourced from Bloomberg):

6-month zero-coupon bonds yielded 0.11% (so r_.

12-month zero-coupon bonds yielded 0.16%.

Find the forward rate from month 6 to 12.

Remember that US (and Australian) bonds normally pay semi-annual coupons.

Answer: Even though these are zero-coupon bonds, since they are US bonds the yield would be quoted as an APR compounding semi-annually since all coupon bonds pay semi-annual coupons.

Therefore we have to convert the APR compounding every 6 months to an effective 6 month yield by dividing it by 2.

$$r_{0-1,eff\ 6mth} = \frac{r_{0-1,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0011}{2} = 0.00055$$

$$r_{0-1,eff\ 6mth} = \frac{r_{0-1,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0016}{2} = 0.0008$$

We want to find $r_{1-2,eff\ 6mth}$, which is the effective 6 month forward rate over the second 6 month period (or 0.5 years to 1 year).

Applying the term structure of interest rates equation:

$$(1 + r_{0-T})^T = (1 + r_{0-1})(1 + r_{1-2})(1 + r_{2-3}) \ldots (1 + r_{(T-1)-T})$$

$$(1 + r_{0-2})^2 = (1 + r_{0-1})(1 + r_{1-2})$$

Note that all rates are 6 month effective rates, so we're using 6-month periods in the indexes $(1 + r_{0-2})^{\mathbf{2}}$, and in the subscript $r_{\mathbf{0-2}}$ where 0-2 denotes the first 2 six-month periods. The subscript is just a label to avoid confusion.

$$(1 + r_{0-2})^2 = (1 + r_{0-1})(1 + r_{1-2})$$

$$(1 + 0.0008)^2 = (1 + 0.00055)(1 + r_{1-2})$$

$$r_{1-2} = \frac{(1 + 0.0008)^2}{(1 + 0.00055)} - 1$$

$$= 0.001050062 = 0.105\%$$

$$r_{1-2} = 0.001050062 = 0.105\%$$

But this is an effective 6 month rate. Let's convert it to an APR compounding every 6 months.

$$r_{1-2,APR\ comp\ semi-annually} = r_{1-2,eff} \times 2$$

$$= 0.001050062 \times 2$$

$$= 0.002100124 = 0.21\%$$

Note that this forward rate APR from 0.5 years to 1 year is bigger than both of the bond yield APR's (which are spot rates).

This makes sense since we have a normal upward sloping yield curve ($r_{0-1} > r_{0-2}$) so the forward rate (r_{1-2}) should be greater than the spot rates ($r_{0-1}\ and\ r_{0-2}$).